SVICI – IZVORI HOMOGENIH MAGNETSKIH POLJA COILS – SOURCES OF HOMOGENEOUS MAGNETIC FIELDS

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Za umjeravanja instrumenata kojima se mjere magnetska polja, kao i za različite pokuse i ispitivanja u elektrotehnici, fizici, medicini, biologiji i ostalim znanstvenim i stručnim područjima, rabe se izvori poznatih homogenih magnetskih polja. Ovisno o vrijednosti željenog magnetskog polja to mogu biti permanentni ili elektromagneti te, češće, različiti svici koji se napajaju istosmjernom, izmjeničnom ili impulsnom strujom. Homogena magnetska polja koja se postižu svicima ili sustavima svitaka u širokom su rasponu od reda veličine mikrotesle, s običnim svicima, do reda veličine deset tesla sa supravodljivim svicima.

The sources of known homogeneous magnetic fields are used for calibration of instruments with which magnetic fields are measured and for various experiments and tests in electrical engineering, physics, medicine, biology and other scientific and technical areas. Depending on the value of the desired magnetic field, these can be permanent magnets or electromagnets and, more frequently, various coils (solenoids) fed by DC, AC or impulse current. The homogeneous magnetic fields attained by coils or coil systems widely vary from the microtesla order of magnitude with ordinary coils to the microtesla order of magnitude with superconductive coils.

Ključne riječi: Helmholtzovi svici, homogeno magnetsko polje, magnetsko polje, svici Keywords: Helmholtz coils, homogeneous magnetic field, magnetic field,

coils



1 UVOD

Svi uređaji, naprave i sustavi za profesionalne i ostale primjene, koji se napajaju električnom energijom, izvori su električnih ili magnetskih polja. Električna i magnetska polja mogu biti istosmjerna ili izmjenična različitih frekvencija, a iznosi im mogu biti u širokom rasponu. Na niskim frekvencijama električna polja u osnovi su razmjerna visini priključenog napona, a magnetska jakosti struje. Oba su polja ovjsna i o obliku i dimenzijama naprave, uređaja ili sustava. Električna i magnetska polja mjere se prikladnim instrumentima i mjernim metodama. Instrumenti koji su namijenjeni za mjerenje magnetskih polja nazivaju se magnetometrima. Rade na različitim načelima te ih danas ima petnaestak vrsta, od jednostavnih svitaka do veoma složenih, npr. s protonskom precesijom. Rabe se u mnogim područjima od fizike do arheologije. Najčešće mjere magnetsku indukciju B (gustoću magnetskog toka), čija je SI jedinica tesla (T). Neki od tih instrumenata umjereni su u staroj jedinici za indukciju, gaus (G). Vrijedi: 1 T = 10⁴ G. Magnetometri se, kao i svi ostali instrumenti, radi provjere ispravnosti i nesigurnosti mjerenja, trebaju povremeno umjeravati.

Umjeravanja se obavljaju izvorima homogenih magnetskih polja poznate indukcije i smjera. Takvi se izvori rabe i za mnoge primjene i pokuse u elektrotehnici, fizici, medicini itd. Pod homogenosti magnetskog polja podrazumijeva se odstupanje vrijednosti magnetske indukcije B u nekoj točki P od njezine vrijednosti B_0 u referentnoj točki, obično središtu svitka, sustava svitaka, elektromagneta ili permanentnih magneta. Homogenost je ovisna o vrsti i dimenzijama svitka, sustava svitaka, ili magneta. Bolja homogenost označava manju promjenu indukcije unutar nekog obujma. Homogenost se iskazuje relativno, ili postotno. Nesigurnost vrijednosti magnetske indukcije u nekoj točki, među ostalima, ovisi o tolerancijama dimenzija svitka i stalnosti struje kroz svitke te smetnjama. Izvori smetnji mogu biti umjetni i prirodni. Umjetni su izvori razne naprave, aparati, sustavi i vodiči kojima teku električne struje, a prirodni izvor je Zemljino magnetsko polje.

U praksi se vrlo često mjere magnetske indukcije reda veličine militesla ili manje. U taj raspon spadaju npr. indukcije u blizini različitih električnih naprava i sustava [1]. Pri preciznijim mjerenjima polja tog reda veličine, poduzimaju se zahvati da se utjecaj smetnji posve otkloni, ili smanji na najmanju moguću mjeru. To se postiže npr. obavljanjem mjerenja u prostoru dovoljno udaljenom od objekata ili područja u kojima se nalaze mogući umjetni izvori smetnji. Najčešće su to priklad-

1 INTRODUCTION

All units, devices and systems for professional and other applications, when fed by electric power, are sources or electrical or magnetic fields. Electric and magnetic fields can be DC or AC of different frequencies, and their amounts can vary across a broad range. At low frequencies the electric fields are basically proportional to the value of connected voltage and the magnetic fields to the strength of current. Both fields also depend on the shape and dimensions of a unit, device or system. Electric and magnetic fields are measured by means of appropriate instruments and measuring methods. The instruments designed for the measurement of magnetic fields are magnetometers. They operate on different principles and are thus available in fifteen or so types, ranging from simple coils to highly complex, e.g., proton procession magnetometers. They find use in areas as diverse as physics and archaeology. What they typically measure is magnetic induction B (magnetic flux density), the SI unit of which is tesla (T). Some of these instruments are calibrated in gauss (G), the old induction unit. It holds: $1 \text{ T} = 10^4 \text{ G}$. Magnetometers, like any other instruments, must be occasionally calibrated to check their reliability and measurement uncertainty.

Calibration is carried out by means of the sources of homogeneous magnetic fields of known induction and direction. Such sources are also used for many applications and experiments in electrical engineering, physics, medicine, etc. Magnetic field homogeneity means the deviation of the magnetic induction value B in a point P from its value B_0 in the reference point, usually the middle of a coil, a coil system, an electromagnet or permanent magnets. Homogeneity depends on the type and dimensions of a coil, a coil system or a magnet. Better homogeneity means less induction changes within a volume. Homogeneity expressed in relative or percental terms. Uncertainty of the magnetic induction value in a point depends, inter alia, on the dimensional tolerances of a coil, the constancy of the current flowing through the coils, and disturbances. The sources of disturbances can be artificial and natural. The artificial sources are various devices, apparatuses, systems and conductors through which electric currents flow, whereas the natural source is the Earth's magnetic field.

In practice very often measured are magnetic inductions of the millitesla or less order of magnitude. That range includes, for example, inductions occurring near various electric devices and systems [1]. For more accurate measurement of the fields of that order of magnitude, steps are taken to eliminate or minimize the effect of disturbances. The way to do it is, for example, taking measurements in places sufficiently distant from the buildings or



ni nemagnetski objekti, obično drveni, nekoliko desetaka ili stotina metara od pretpostavljenih izvora smetnji na tlu bez tragova feromagneskih tvari. Unatoč tomu i u takvom okolišu ponekad se trebaju te smetnje kompenzirati.

2 ZEMLJINO MAGNETSKO POLJE

O izvoru Zemljinog magnetskog polja (geomagnetsko polje) još postoji mnogo nepoznanica, jer unutrašnjost Zemlje nije dovoljno poznata. Magnetsko polje na površini Zemlje sastoji se od dva dijela [2], [3] i [4]. Prvi, i to glavni dio, stvaraju, prema najčešće spominjanoj teoriji, električne struje duboko u unutrašnjosti Zemlje (dinamo učinak) te magnetske stijene i električne struje u Zemljinoj kori. Struje u Zemljinoj dubini posljedica su gibanja tekućeg željeza s dodatkom nikla (temperature 4 800 °C), u tzv. vanjskoj jezgi, na dubini od, približno, 3 000 km do 5 000 km, između središnie čvrste metalne iezare i tzv. donjeg plašta, gornjeg plašta i kore. Razvidno je da se ta teorija teško može dokazati mjerenjima i pokusima. Glavno polje nije stalno. Dugoročna njegova promjena naziva se sekularnom varijacijom. Polje nastalo od magnetskih stijena i struja u Zemljinoj kori, relativno je slabo i vremenski stabilno. Drugi dio Zemljinog magnetskog polja nastaje od električnih struja u atmosferi i oko nje. Taj dio polja pod utjecajem je Sunca [5] i vremenski se naglo mijenja.

Zemlja djeluje kao magnetski dipol. Sjeverni i južni magnetski polovi ne podudaraju se s istoimenim geografskim polovima. Magnetski polovi, neovisno jedan o drugome, vremenski dugoročno, mijenjaju svoje položaje prema istoimenim geografskim polovima. Zemljino magnetsko polje F, najčešće se iskazuje u nanoteslama, može se rastaviti u komponente: vertikalnu Z (u literaturi se označava i sa V), usmjerenu prema tlu, i horizontalnu H (slika 1) [6]. Potonja ima sjevernu komponentu X s pozitivnim predznakom kada je usmjerena prema geografskom (pravom) sjeveru i istočnu komponentu Y s pozitivnim predznakom u smjeru istoka. Osim navedenih podataka još su važni kut između Xi H, tzv. deklinacija ili varijacija D, između geografskog i magnetskog sjevernog pola, s pozitivnim predznakom u smjeru istoka te kut između H i F, iznad ili ispod horizonta tzv. inklinacija ili dip I, s pozitivnim predznakom prema dolje. Deklinacija i inklinacija iskazuju se u stupnjevima i minutama. Ove dvije veličine posebice su važne za navigaciju uz pomoć magnetskog kompasa. Nepoznate komponente Zemljinog magnetskog polja mogu se odrediti iz tri poznate. Tako je npr. $D = \arctan(Y/X), Y = H \sin D$ itd. 0 vremenskim i areas with potential artificial sources of disturbances. Such places are usually suitable non-magnetic structures, usually wooden, some tens or hundreds meters away from the assumed sources of disturbances on the ground without traces of ferromagnetic materials. In spite of it, in such an environment these disturbances should sometimes be compensated.

2 EARTH'S MAGNETIC FIELD

There are still guite a lot of unknown facts about the source of the Earth's magnetic field (geomagnetic field), because the Earth's interior is insufficiently known. Magnetic field at the surface of the Earth consists of two parts [2], [3] and [4]. The first and the main part is generated, according to the most guoted theory, by electric currents deep inside the interior of the Earth (dynamo effect), as well as by magnetic rocks and electric currents in the Earth's crust. Currents in the depth of the Earth are a result of the motion of liquid iron with addition of nickel (temperature 4 800 °C) in the outer core at a depth of 3 000 km to 5 000 km, between the central firm metal core and the lower mantle, upper mantle and crust. It is obviously hard to prove this theory by measurement and experimentation. The main field is not constant. Its long-term change is referred to as secular variation. The field generated by magnetic rocks and electric currents in the Earth's crust is relatively weak and stable over time. The second part of the Earth's magnetic field is generated by electric currents in the atmosphere and around. That part of the field is under the solar influence [5] and is rapidly changing over time.

The Earth acts as a magnetic dipole. The north and south magnetic poles do not correspond with the geographic poles. The magnetic poles change their positions on a long-term basis and independent of each other in relation to their geographic counterparts. The Earth's magnetic field F most often expressed in nanoteslas, can be divided into the following components: the vertical Z (in reference literature also indicated V, directed to the ground, and the horizontal component H (Figure 1) [6]. The latter has the north component X with positive sign when directed to the geographic (real) north and the east component Y with positive sign directed to the east. In addition to the mentioned values, also important are the angle between X and H, the declination or variation D_{1} between the geographic and magnetic north poles, with positive sign in the direction of east, and the angle between H and F, above or below the horizon, the inclination or dip I, with positive sign downward. The declination and inclination are expressed in degrees and minutes. These two values are particularly important for navigation by means of the magnetic compass. The prostornim promjenama nadležne ustanove (geomagnetski opservatorij) u određenim vremenskim razmacima objavljuju podatke. unknown components of the Earth's magnetic field can be determined from the three known components. For example, $D = \arctan(Y / X)$, $Y = H \sin D$ etc. At certain intervals competent establishments (geomagnetic observatories) release data on spatiotemporal changes.



Slika 1 – Sastavnice Zemljinog magnetskog polja [6] Figure 1 – Components of the Earth's magnetic field [6]

Magnetska indukcija na površini Zemlje, ovisno zemljopisnoj širini i duljini u rasponu je, približno, od 30 000 nT do 60 000 nT, odnosno od 30 μ T do 60 μ T. Istraživanja Zemljinog magnetskog polja počela su već u 13. stoljeću nakon što se u Europi kompas počeo upotrebljavati u 12. stoljeću. Sustavna mjerenja Zemljinog magnetskog polja na tlu započela su 1840. godine, a u prošlom stoljeću i iz zraka niskoletećim zrakoplovima te kasnije sa satelita. Valja naglasiti da Zemljino magnetsko polje čuva Zemlju od bujica plazme koja dolazi sa Sunca [5].

Radi dobivanja uvida u konkretne podatke, evo rezultata mjerenja Zemljinog magnetskog polja u više točaka, obavljenog sjeveroistočno i istočno od Zagreba, u drugoj polovini 2003. godine. Izmjerena magnetska indukcija na tom području bila je u rasponu od 47,3 μ T do 47,7 μ T. Usporedbom s mjernim rezultatima iz 1927. godine na istom području ustanovljena je promjena od +40 nT godišnje [7]. Zapaženi su prostorni gradijenti od 18 nT/10 km do 37 nT/10 km te trenutačne promjene čija je srednja vrijednost približno 2 nT tijekom 15 minuta.

Smanjenje mjerne nesigurnosti zbog utjecaja Zemljinog magnetskog polja, ali i drugih izvora, pri proizvodnji, mjerenju, umjeravanju itd. magnetskih polja indukcije reda veličine 1 mT ili manjih, postiže se kompenzacijom jedne ili, češće, svih komponenti smetajućeg polja. Za kompenzaciju najčešće se rabe tri para Helmholtzovih svitaka čije se struje automatski ugađaju po veličini The magnetic induction at the surface of the Earth, in dependence on the geographic latitude and longitude, varies approximately from 30 000 nT to 60 000 nT, or from 30 μ T to 60 μ T. Research in the Earth's magnetic field started back in the 13th century after the introduction of the mariner's compass in the century before. Systematic measurements of the Earth's magnetic field started in 1840, in the last century from low-flying aircraft, later from satellites. It should be noted that by its magnetic field the Earth is shielded from the solar winds [5].

As a concrete example, below given are the results of the measurements of the Earth's magnetic field in more points, made northeast and east of Zagreb in the second half of 2003. The measured magnetic induction in that area varied from 47,3 μ T to 47,7 μ T. A comparison with the measurement results from 1927 in the same area showed a change of +40 nT per year [7]. Space gradients from 18 nT/10 km to 37 nT/10 km were noticed, as well as instantaneous changes with the mean value of about 2 nT over 15 minutes.

A reduction in the measurement uncertainty caused by the influence of the Earth's magnetic field and other sources in the production, measurement, calibration etc. of magnetic induction fields of 1 mT order of magnitude is achieved by compensating one or, more often, all components of the disturbing field. Used most often for compensation are three pairs of the Helmholtz coils, the currents of which are automatically tuned by size and direction, so that in size the fields are the same, in direction



i smjeru, tako da su polja po veličini ista, a po smjeru suprotna smetajućim poljima [8], koja se mjere osjetljivim magnetometrima.

3 IZVORI HOMOGENIH MAG-NETSKIH POLJA

Homogonena magnetska polja mogu se ostvariti permanentnim magnetima, elektromagnetima, svicima (zavojnice, solenoidi) ili sustavom svitaka, kojeg čine najmanje dva svitka, kojima teku električne struje. Permanentni magneti ili elektromagneti, kao i supravodljivi svici rabe se za indukcije od reda veličine 10^{-2} T sve do reda veličine 10 T. Takve se vrijednosti indukcija pretežito rabe u fizici, medicini (npr. za magnetsku rezonanciju-MRI) itd. Manje indukcije od reda veličine 10⁻² T ostvaruju se svitkom ili sustavom svitaka, a vrijednosti indukcije ugađaju se u širokom rasponu regulacijom jakosti struje. Raspon ugađanja magnetskog polja ovisi o vrsti i dimenziji svitka, ili sustava svitaka, promjeru vodiča, vrsti hlađenja itd.

Ovisno o ispitivanom objektu rabe se zračni valjkasti jednoslojni svici, ali i oni višeslojni, ili sustavi svitaka poznati kao Helmholtzovi svici, Maxwellovi svici itd. Za tijela svitaka rabe se različiti materijali, čije dimenzije trebaju biti što manje vremenski i temperaturno ovisni te bez tragova feromagnetskih tvari. Kakvoću svitka, među ostalim, označava konstanta svitka iskazana u [T/A], tj. kolika se indukcija postiže, iskazana u teslama, za određenu jakost struje iskazanu u amperima. Relativna nesigurnost te konstante, a to znači i postignute indukcije, može biti i reda veličine 10⁻⁶.

Udaljavanjem, u radijalnom i aksijalnom smjeru, od središnje točke svitka, ili sustava svitaka, vrijednost magnetske indukcije se mijenja ovisno o vrsti i dimenzijama svitka. U praksi je, za umjeravanje i ispitivanja utjecaja magnetskog polja na neke objekte, u pravilu zanimljiva magnetska indukcija u središtu svitka ili blizu njega i to u aksijalnom smjeru, koja se najčešće označava kao z os. Razlog tomu je što se objekt koji se podvrgava utjecaju magnetskog polja stavalja u središte svitka ili sustava svitaka.

3.1 Valjkasti svici (zavojnica, solenoid)

Ako ispitivani objekt u homogenom magnetskom polju, za vrijeme pokusa, ne treba biti vidljiv, jer se na njemu ne treba očitavati odziv na polje, za stvaranje polja često se rabi valjkasti jednoslojni svitak. Za tijelo svitka u obliku šupljeg valjka rabe se različiti materijali od drveta do kremena (kvarca). Potonji se rabi za vrhunska mjerenja, contrary to the disturbing fields [8] which are measured by sensitive magnetometers.

3 SOURCES OF HOMOGENEOUS MAGNETIC FIELDS

Homogeneous magnetic fields can be generated by permanent magnets, electromagnets, coils or a coil system composed of at least two coils through which electric currents flow. Permanent magnets or electromagnets, as well as coils, are used for inductions of 10^{-2} T order of magnitude and up to 10 T. Such induction values are largely used in physics, medicine (magnetic resonance imaging – MRI), etc. Smaller inductions of 10^{-2} T order of magnitude are generated by a coil or a coil system and the induction values are tuned across a broad range by regulating the strength of current. The tuning range of a magnetic field depends on the type and size of the coil, the conductor diameter, type of cooling, etc.

Depending on the tested object, in use are onelayer roller air coils, as well as multi-layer coils or coil systems known as Helmholtz coils, Maxwell coils, etc. For the coil bodies various materials are used with dimensions that should be time and temperature dependent to the least possible degree and without any traces of ferromagnetic materials. The coil quality is indicated, among other things, by the coil constant expressed in [T/A], i.e., how much induction is attained, expressed in tesla unit, for a certain strength of current expressed in amperes. The relative uncertainty of that constant, and thereby of the attained induction, may well reach 10⁻⁶ order of magnitude.

With the movement in radial or axial direction away from the central point of a coil or a coil system, the value of magnetic induction is changing in dependence on the type and dimensions of the coil. What is interesting in practice for calibration and testing of the impact of magnetic field on some objects is as a rule the magnetic induction in the middle of the coil or close to it in axial direction, usually indicated as *z* axis. The reason is that the object subjected to the influence of a magnetic field is placed in the middle of the coil or coil system.

3.1 Roller coils

If a tested object in a homogeneous magnetic field need not be visible during the test, because the field response need not be read on it, then a one-layer roller coil is often used for field generation. For the hollow roller shaped body of the coil various materials are used, from wood to quartz, the latter for high-order measurements, such as those aimed to determine certain physical constants. The outer surface of such a hollow roller is finely polished kao što su ona za određivanje pojedinih fizikalnih konstanti. Vanjska se površina takovog šupljeg valjka fino polira s preciznošću reda veličine mikrometra. Zatim se na njoj gusto i precizno ureže plitka helikoida za svitak, tako da su zavoji blizu jedan do drugoga. Prije i nakon obrade precizno se određuju dimenzije valjka na temelju kojih se proračunava vrijednost indukcije. Namatanje s određenom silom jednoznačno određuje položaj svakog zavoja svitka, koji se laserski provjerava s nesigurnošću reda veličine mikrometra [8], [9] i [10].

Jeftiniji postupak, ali stoga i manje pouzdan kod zagrijavanja svitka Jouleovom toplinom, temelji se na namatanju u prostoriji zagrijanoj na temperaturu od 30 °C do 35 °C, a mjerenja se obavljaju u prostoru na temperaturi oko 23 °C. Time se postiže da su zavoji učvršćeni u svojem položaju silom nastalom skraćivanjem zavoja zbog snižavanja temperature.

Potrebni se broj zavoja namata kalibriranom bakrenom žicom velike čistoće (99,99 %), ili žicom od posebne slitine velike vodljivosti, kako bi se postigla što veća gustoća struje. Ako žica nije izolirana tada je razmak između zavoja reda veličine milimetra. Promier žice odabire se ovisno o jakosti struje koja će njome teći, odnosno o najvećoj indukciji koja se želi postići u određenom opsegu u središtu svitka. Pri tome se vodi računa da polumjer žice bude zanemariv prema polumjeru tijela svitka, a duljina svitka višestruko veća od njegova polumjera, kako bi homogeno polje zauzimalo što veći postotak duljine svitka. Razvijena Jouleova toplina zbog struje kroz svitak ne smije bitno utjecati na promjenu njegovih dimenzija. Svi se spomenuti postupci provode kako bi se stvarni svitak približio idealnom, odnosno proračunata magnetska indukcija u predviđenom opsegu unutar svitka bude jednaka stvarnoj.

Magnetsko polje, odnosno magnetska indukcija, svitka računa se pomoću magnetskog polja u točki P na osi z kružne strujnice polumjera *a*, koja se nalazi u ravnini *xy*. Strujnicom teče struja *I*. Neka je točka P na udaljenosti *r* od malog elementa *dI* strujnice, u_t jedinični vektor u smjeru struje tangecijalno na element *dI*, a u_r jedinični vektor usmjeren od elementa *dI* prema točki P. Prema Biot-Savartovom zakonu magnetska indukcija u točki P je: with micrometer precision. Then a shallow helicoid for the coil is carved in the surface densely and with precision, so that turns are close to each other. Before and after the treatment the roller dimensions are accurately determined, based on which the induction value is computed. Winding with a certain force unequivocally defines the position of each turn of wire, which is laser-checked within micrometer uncertainty margin [8], [9] and [10].

A more economical, but less reliable procedure involving the heating of a coil as a result of the Joule effect, is based on winding in a room heated to a temperature of 30 °C to 35 °C, with measurement being performed in a room heated to about 23 °C. This ensures that the turns of wire are fixed in their positions by the force produced by wire contraction due to falling temperature.

The required number of turns is wound with a calibrated copper wire of high purity (99,99 %), or a wire from special high-conductive alloy in order to achieve maximum current density. If the wire is not insulated, the distance between the turns of wire is within a millimeter margin. The diameter of the wire is selected in dependence on the strength of the current that will flow through it, or in dependence on maximum induction to be achieved within a certain range in the middle of the coil. It must be made sure that the wire diameter is negligible in relation to the radius of the coil body and that the length of the coil is several times greater than its radius, so that the homogeneous field may cover the highest possible percentage of the coil length. The Joule warmth generated by the passage of current through the coil must not significantly affect the size of the coil. All the mentioned procedures are carried out for the real coil to approximate the ideal coil, or for the calculated magnetic induction to equal the real induction across the planned range within the coil.

The magnetic field or magnetic induction of a coil is computed by means of the magnetic field in point P on axis z of the circular current loop with radius a on plane xy. Current I flows through the current loop. Let point P be at a distance r from the small element dI of the current loop, u_t unit vector in the direction of the current tangentially to the element dI, and u_r unit vector directed from the element dI towards point P. According to the Biot-Savart Law, magnetic induction in point P is:

$$\boldsymbol{B} = \frac{\mu_0}{4\pi} I \oint \frac{\boldsymbol{u}_{\mathrm{t}} \mathbf{x} \boldsymbol{u}_r}{\boldsymbol{r}^2} \mathrm{d}l ,$$

(1)

gdje je $\mu_0 = 4\pi \cdot 10^{-7}$ Vs/Am permeabilnost praznine (vakuuma).

where $\mu_{\rm 0} = 4\pi \, \cdot \, 10^{\text{-7}}$ Vs/Am void (vacuum) permeability.



Računanje integrala u (1) za bilo koju točku je složeno [11] i [12]. Međutim za točku P na osi *z* je rješenje jednostavno: Integration in (1) for any point is complex [11] and [12]. However, for point P on axis z the solution is simple:

$$B_{z} = \frac{\mu_{0} I a^{2}}{2 \left(a^{2} + z^{2} \right)^{3/2}} .$$
 (2)

Jednoslojni se svitak može smatrati sastavljenim od niza strujnica na zajedničkoj osi, kojima teče ista struja (slika 2). A one-layer coil can be considered composed of a series of current loops on the common axis through which the same current flows (Figure 2).



Slika 2 – Jednoslojni svitak (zavojnica, solenoid) Figure 2 – One-layer coil

Ukupna magnetska indukcija na osi *z* dobiva se zbrajanjem polja pojedinih strujnica. Neka je broj zavoja *N* svitka velik, a *I* njegova ukupna duljina. Uz pretpostavku da je polumjer žice zanemariv u usporedbi s polumjerom zavojnice *a* i da je gustoća strujnica *n* po jedinici duljine *N/I*, dio svitka dz ima $(N/I) \cdot dz$ strujnica. Iz (2) slijedi da taj dio svitka u točki P stvara magnetsku indukciju: Total magnetic induction on axis z is obtained by summing up the fields of individual current loops. Let the number N of turns be great, and I the coil's total length. Given a negligible wire radius compared with coil radius a and the flux density n per unit of length N/I, the part of the coil dz has $(N/I) \cdot$ dz current loops. It follows from (2) that this part of the coil in point P generates magnetic induction:

$$dB_{z} = \frac{\mu_{0} I n a^{2} dz}{2 (a^{2} + z^{2})^{3/2}}$$
 (3)

Zamijeni li se varijabla z sa \hat{a} te integriranjem od \hat{a} , do \hat{a} , dobiva se:

If variable z is substituted by \hat{a} and through integration from \hat{a}_1 to \hat{a}_2 we get:

$$B_{z} = \frac{\mu_{0} nI}{2} \left[\frac{z+c}{\sqrt{(z+c)^{2}+a^{2}}} - \frac{z-c}{\sqrt{(z-c)^{2}+a^{2}}} \right],$$
(4)

gdje je c = l/2, odnosno 2c = l.

where c = l/2 or 2c = l.

U središtu svitka je z = 0, pa se za magnetsku indukciju dobiva: In the middle of the coil it is z = 0, so for magnetic induction we get:

$$B_0 = \frac{\mu_0 nI}{2} \frac{2c}{\sqrt{c^2 + a^2}} = \frac{\mu_0 NI}{l} \frac{c}{\sqrt{c^2 + a^2}} \quad , \tag{5}$$

Za c > aslijedi da je $B_0 \approx i_0 \cdot n \cdot I = i_0 \cdot I \cdot N/I$, a na krajevima svitka gdje je z = c, dobiva se da je $B_z \approx B_0/2$, tj. indukcija je na krajevima svitka dva puta manja od one u njegovom središtu.

Želi li se postići homogeno polje u što većem opsegu omjer l/a mora biti što veći. Za različite fizikalne pokuse, npr. određivanja nekih konstanti. potrebni su svici u kojima je relativna homogenost magnetskog polja u određenom opsegu reda veličine 10^{-7} . To se može postići svicima čija je duljina reda veličine 10³ metara, što je tehnički i ekonomski neizvedivo. Stoga se poduzimaju različiti zahvati s ciliem da se takva homogenost postigne. u opsegu kugle polumjera nekoliko desetaka milimetra, svitkom duljine od jednog do dva metra promjera reda veličine 0,1 m. Svitak se sastoji iz više dijelova kroz koji teku struje različitih jakosti i smjerova, ili se preko temeljnog svitka namota drugi višedijelni kojim teku struje različitih jakosti [8] i [9].

Osim jednoslojnih rabe se, posebice za veće magnetske indukcije, višeslojni valjkasti svici. Proračun magnetske indukcije za bilo koju točku unutar ili izvan takvog svitka je složen, slično kao i za jednoslojni svitak [11] i [13]. Za indukciju za bilo koju točku na osi z višeslojnog svitka (slika 3) izraz za indukciju je jednostavniji [14]: For c > a it follows that $B_0 \approx i_0 \cdot n \cdot l = i_0 \cdot l \cdot N/l$, whereas at the coil ends, where z = c, we get $B_z \approx B_0/2$, i.e., induction at the coil ends is two times smaller than the one in the middle of the coil.

If a homogeneous field is to be achieved to maximum extent, the ratio l / a must be as great as possible. For various physical experiments, such as determination of some constants, coils are needed in which the relative homogeneity of magnetic field within a certain scope is of 10-7 order of magnitude. This can be achieved by coils with lengths of 10^3 meter order of magnitude, which is technically and economically unfeasible. For that reason various attempts are being made aimed to achieve such homogeneity within the range of a sphere having a radius of some tens of millimeters, with a coil 1 m to 2 m long, diameter 0,1 m. The coil consists of more parts through which electric currents of varying strengths and directions are flowing, or another multi-part coil is wound over the main coil to take on currents of varying strengths [8] and [9].

Apart from the one-layer coils, the multi-layer roller coils are also used, especially for higher magnetic inductions. The computation of magnetic induction for any point inside or outside of such a coil is a complex procedure, just as it is for a one-layer coil [11] and [13]. The expression for induction at any point on axis z of a multi-layer coil (Figure 3) is simpler [14]:

$$B_{z} = \frac{\mu_{0}nI}{2(a_{2}-a_{1})} \left[(z+c)\ln\frac{\sqrt{a_{2}^{2}+(z+c)^{2}}+a_{2}}{\sqrt{a_{1}^{2}+(z+c)^{2}}+a_{1}} - (z-c)\ln\frac{\sqrt{a_{2}^{2}+(z-c)^{2}}+a_{2}}{\sqrt{a_{1}^{2}+(z-c)^{2}}+a_{1}} \right],$$
(6)

gdje su a_1 unutarnji, a a_2 vanjski polumjer višeslojnog svitka duljine $l = 2 \cdot c$.

where a_1 is the inner and a_2 the outer radius of a multi-layer coil with length $l = 2 \cdot c$.

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U središtu svitka, tj. za z = 0, indukcija je:

In the middle of the coil, i.e., for z = 0, induction is:

$$B_0 = \frac{\mu_0 NI}{2(a_2 - a_1)} \ln \frac{\sqrt{a_2^2 + c^2} + a_2}{\sqrt{a_1^2 + c^2} + a_1}$$
(7)

3.2 Kratki svici

Postoje niz pokusa i mjerenja u kojima objekt koji se ispituje treba biti lako dostupan i vidljiv. To je npr. slučaj kod provjere utjecaja magnetskog polja na pokazivanje analognih ili digitalnih mjernih instrumenata. Tada se na instrumentu očitava moguća razlika u pokazivanju pri isključenom i uključenom magnetskom polju određene indukcije. Stoga se, za te i slične primjene, magnetsko polje stvara jednim kratkim svitkom ili sustavima s dva ili više kratkih svitaka.

3.2.1 Jednostavni kratki svitak

Kratki svitak može biti kružni ili pravokutni. Potonji je jednostavniji za izradu, pa se stoga češće rabi. Za tijelo svitka rabe se različiti materijali, ali preteže drvo. Njegova duljina (visina) obično je manja 0,1 *a*.

Indukcija u središtu kratkog kružnog svitka sa Nzavoja, tj. svitka kod kojeg je c > a, a zanemarive površine presjeka, prema (5) je:

3.2 Short coils

There are many experiments and measurements where the tested object must be easily accessible and visible, for example, when the influence of magnetic fields on the display of analog or digital measuring instruments is checked. Then the instrument possibly displays a difference in the turned-off and turned-on magnetic field of a certain induction. Thus for these and similar applications the magnetic field is generated by a short coil or by systems of two or more shorts coils.

3.2.1 A simple short coil

A short coil can be circular or rectangular. The latter is easier to make and thus more often used. Various materials are used for the coil body, but wood prevails. Its length (height) is usually less than 0,1 a.

Induction in the middle of a short circular coil with N turns, i.e., the coil where c > a, and with negligible cross-section surface, is according to (5):

$$B_0 = \frac{\mu_0 NI}{2a} \cdot$$

(8)

Za umjeravanje mjerila magnetske indukcije niske frekvencije međunarodna norma [15] preporučuje pravokutni svitak stranica 2a i 2b sa N zavoja. InFor the calibration of low-frequency magnetic induction meters the international standard [15] recommends a rectangular coil with sides 2a and

dukcija u bilo kojoj točki P(x, y, z) u prostoru dana je složenim izrazom i zato se ovdje ne navodi, jer je u praksi zanimljiva indukcija u ravnini svitka, gdje se stavlja ispitivan senzor polja obično malih dimenzija. Uz pretpostavku da vodiči svitka imaju zanemarivu površinu presjeka, indukcija u smjeru osi z u središtu kvadratnog svitka duljine stranica 2a, iskazanoj u metrima, kojim teče struja I iskazanu u amperima je [15]: 2b and with *N* turns. Induction in any point P(*x*, *y*, *z*) in space is given in a complex expression and hence not stated here, because what is of interest in practice is induction in the plane of the coil where a usually small-sized tested field sensor is placed. Assuming that the coil conductors have a negligible cross-section surface, induction in the direction of *z* in the middle of a square coil with the side length 2a, expressed in meters, through which current *I* flows, expressed in amperes, [15] amounts to:

$$B_0 = \frac{\mu_0 I N \sqrt{2}}{\pi a}$$
 (9)

Kako se najčešće rabi takav svitak dimenzija 1 m x 1 m (2a = 1 m), na slici 4 prikazana su izmjerena postotna odstupanja indukcije u točkama blizu središta svitka od one u središtu, mjernim svitkom promjera 0,10 m, u ravnini svitka. U zagradama su odstupanja 0,03 m iznad i ispod ravnine svitka [15]. Nesigurnost vrijednosti indukcije procjenjuje se ± 1,1 %. As such a coil sized 1 m x 1 m (2a = 1m) is widely used, Figure 4 shows percental deviations of induction in points close to the middle of the coil from that in the middle, measured by a measuring coil of dia. 0,10 m, in the plane of the coil. Stated in brackets are =0,03 m deviations above and below the plane of the coil [15]. The uncertainty of the induction value is assessed at ± 1,1 %.



 $\begin{array}{l} \textbf{Slika 4} - \textbf{Izmjerena postotna odstupanja indukcije B}_{\text{F}} [15] \\ \textbf{Figure 4} - \textbf{Measured percental deviations of induction B}_{\text{F}} [15] \end{array}$

3.2.2 Helmholtzovi svici

Sustav s dva jednaka kratka kružna ili pravokutna svitka, koji su serijski povezana, međusobno razmaknuta na određenu udaljenost, kojima teče ista struja, nazivaju se Helmholtzovim svicima u čast njemačkog liječnika i fizičara H.L.F. Helmholtza (1821. – 1894.). On je dao veliki doprinos u razvoju obje struke. Njegovi su učenici bili, kasnije poznati fizičari, među ostalima, Hertz, Pupin i Schottky.

U prostoru između svitaka postoji homogeno magnetsko polje u smjeru osi svitaka. Indukcija, koja je zbroj indukcija oba svitka, s udaljavanjem od središta prostora među svicima, manje se mijenja

3.2.2 Helmholtz coils

A system with two equal short circular or rectangular coils, serially connected, set apart at a certain distance, through which the same current flows, are referred to as Helmholtz coils in honor of the German physician and physicist H.L.F. Helmholtz (1821-1894), who significantly contributed to both disciplines. Some of his disciples, including Hertz, Pupin and Schottky, later became renown physicists.

In the space between the coils there is a homogeneous magnetic field in the direction of the axis of the coils. Induction, which is a sum of induc-



nego kod jednog kratkog svitka. Postoji određena razlika u vrijednostima indukcije između kružnih i pravokutnih svitaka, no potonji su lakši za izradu. Ovisno o objektu podvrgnutom ispitivanju, Helmholtzovi svici mogu biti polumjera, odnosno duljine stranica od reda veličine centimetra do reda veličine metra. Time se postižu jednolika polja u vrlo malom opsegu sve do onih većih od 1 m³. Za tijela svitaka rabe se drvo, plastični materijali i metali, najčešće aluminij. Kod potonjeg se mora paziti da ne stvara kratko spojeni zavoj.

Magnetska indukcija u središnjoj točki na osi z između dva kružna svitka sa po N zavoja, svaki polumjera a, zanemarive površine presjeka, kojima u istom smjeru teče struja I, a međusobno su razmaknuti na udaljenost a (slika 5) može se odrediti iz (2) ako se uvrsti z = a / 2: tions from both coils, changes less while moving away from the center of the space in between the coils than it does with a single short coil. There is a certain difference in induction values between the circular and rectangular coils, but the latter are easier to make. Depending on the tested object, the Helmholtz coils can have radii or side lengths in the order of magnitude from one centimeter to one meter. Thereby uniform fields are obtained within a very small range up to those exceeding 1 m³. Used for the coil bodies are wood, plastic materials and metals, mostly aluminum. In the latter case it must be made sure that no short circuits are established.

Magnetic induction in the central point on axis z between two circular coils with N turns each, radius a each, of negligible cross-section surface, through which current I flows in the same direction, spaced at a distance a (Figure 5), can be determined from (2) if z = a / 2 is substituted:



Slika 5 — Helmholtzov sustav s kružnim svicima Figure 5 — Helmholtz system with circular coils

$$B_{0} = \frac{\mu_{0}NIa^{2}}{2} \left\{ \frac{1}{\left[a^{2} + \left(\frac{a}{2}\right)^{2}\right]^{\frac{3}{2}}} + \frac{1}{\left[a^{2} + \left(\frac{a}{2}\right)^{2}\right]^{\frac{3}{2}}} \right\} = \frac{\mu_{0}NIa^{2}}{\left[a^{2} + \left(\frac{a}{2}\right)^{2}\right]^{\frac{3}{2}}}$$
(10)

Nakon sređivanja dobva se:

After reduction we get:

$$B_0 = \frac{\mu_0 NI}{a} \frac{8}{5\sqrt{5}} = 0.715542 \frac{\mu_0 NI}{a} \,. \tag{11}$$

Umjesto kružnih češće se rabe kvadratični svici duljine stranica 2a na međusobom razmaku d(slika 6). Instead of the circular coils, preferably used are the square coils with the length of the sides 2a at a mutual distance d (Figure 6).



Slika 6 — Helmholtzov sustav s kvadratnim svicima Figure 6 — Helmholtz system with square coils

Magnetska indukcija na osi *z* je [16]:

Magnetic induction on axis *z* is [16]:

$$B_{z} = \frac{2\mu_{0}NIa^{2}}{\pi} \left\{ \frac{1}{\left[a^{2} + \left(z - \frac{d}{2}\right)^{2}\right]}\sqrt{2a^{2} + \left(z - \frac{d}{2}\right)^{2}} + \frac{1}{\left[a^{2} + \left(z - \frac{d}{2}\right)^{2}\right]}\sqrt{2a^{2} + \left(z - \frac{d}{2}\right)^{2}} \right\}} \right\}$$
(12)

Radi dobivena maksimuma druga derivacija tog izraza izjednači se s nulom, pa slijedi:

Due to the obtained maximum, the second derivation of that expression is zeroed, so it follows:

$$\frac{d}{2} = 0,5445a$$
. (13)

Dakle, razmak između dva kvadratična svitka u Helmhotzovom sustavu nešto je veći od onog kod kružnih.

Zanimljive su usporedbe zadnja četiri spomenuta izvora magnetskih polja, tj. kružnog i kvadratnog kratkog svitka te kružnih i kvadratičnih Helmholtzovih svitaka.

Na slici 7 [17] prikazana su postotna odstupanja indukcije B_z uzduž osi z od vrijednosti B_0 u središtu kružnog i kvadratnog kratkog svitka (z = 0) te Helmholtzovih sustava s kružnim i kvadratnim svicima kao funkcije od z iskazanih u postocima polumjera a kružnog svitka i polovini duljine stranica kvadratnog svitka. Therefore, the distance between two square coils in the Helmhotz system is a little greater than that between the circular coils.

Quite interesting are comparisons of the last four mentioned sources of magnetic fields, the circular and square short coils and the circular and square Helmholtz coils.

Figure 7 [17] shows percental deviations of induction B_z along axis z from value B_0 in the center of the circular and square short coils (z = 0) and the Helmholtz systems with circular and square coils as a function of z expressed in the percentages of radius a of the circular coil and a half of the side length of the square coil.







Položaj na osi z iskazan je postotkom polumjera a kružnih, odnosno polovini duljine stranice a kvadratnih svitaka. Odstupanje indukcije u smjeru osi z uzduž osi x, za z = 0, prikazana je na slici 8 [15] i [17]. Na apscisi su postoci polumjera kružnog, odnosno polovice duljine stranica kvadratnog svitka. Krivulje su normalizirane radi lakše usporedbe. Iz tih podataka slijedi da je kvadratni kratki svitak bolji je od kružnog. Također su i kvadratni Helmholtzovi svici bolji od kružnih. Kako se iz slika 7 i 8 može vidjeti, razlika kod Helmholtzovih svitaka, kružnih i kvadratnih, nije velika.

Tijekom vremena nastala su različita poboljšanja

Position on axis z is expressed by the percentage of radius a of the circular coils or a half of the side length a of the square coils. Induction deviation in the direction of axis z along axis x for z = 0 is shown in Figure 8 [15] and [17]. On the abscise are the percentages of the circular coil and the halves of the side length of the square coil. The curves are normalized for easier comparison. What follows from these data is that the short square coil is better than the circular one. Likewise, the square Helmholtz coils are better than the circular ones. As shown in Figures 7 and 8, with the Helmholtz coils there is no great difference between the circular and square coils.





temeljnih sustava svitaka. Neka od njih su, radi postizanja bolje homogenosti u većem opsegu, patentirana. Najčešće se to postiže umetanjem jednog ili više svitaka, istih ili različitih dimenzija i broja zavoja između temeljna dva svitka. Tako se npr. između Helmholtzovih svitaka stavlja još jedan par različitih dimenzija, tzv. unutarnjih i vanjskih, kružnih, paravokutnih ili kvadratnih svitaka postiže relativna homogenost reda veliWith the passage of time the basic coil systems have undergone various improvements, some of them patented owing to the attainment of better homogeneity over a wider range. This is typically done by inserting one or more coils of the same or different dimensions and number of turns between the two basic coils. Thus between the Helmholtz coils one more pair of coils of different dimensions, the inner and outer, the circular and rectangular or square Poboljšanje se može postići i npr. dodatkom jednog svitka između dva Helmholtzova svitka [19]. Dodatni svitak istog je polumjera *a* kao i Helmholtzovi svici i udaljen od svakoga od njih 0,762 · *a*, dok je omjer zavoja dodatnog svitka prema vanjskima k = 0,531 5. Indukcija u središtu tog sustava, tj. u z = 0, ako vanjski svici imaju po 111 zavoja, a središnji 59 zavoja je:

Usporedba poboljšanih Helmholtzovih svitaka s

 $B_0 = 0.7704 \frac{111\mu_0 I}{a} \; .$

ones, is inserted and thus the relative homogeneity of 10⁻⁴ order of magnitude is attained [18].

An improvement can also be achieved by adding a coil between two Helmholtz coils [19]. The additional coil has radius *a* like Helmholtz coils and its distance from each of them is $0,762 \cdot a$, whereas the turn ratio of the additional coil to the outer coils is k = 0,531 5. Induction in the middle of this system, i.e., in z = 0, if the outer coils have each 111 turns and the central coil 59 turns of wire, is:

(14)

onimobičnim, kaoi s Maxwellovim svicima grafički je prikazana na slika 9 [19], gdje je $a = (B_z - B_0) / B_0$ relativno odstupanje indukcije po osi z od one u središtu sustava (homogenost), a udaljenost na osi z iskazana je omjerom $\xi = z / a$. Postignuto je bitno poboljšanje homogenosti u većem opsegu (usporedba krivulja 1 i 3).

U boljim laboratorijima i za složenija ispitivanja rabe se temeljni sustavi s najmanje tri para Helmholtzovih svitaka, kako bi se dobilo homogeno magnetsko polje u tri smjera.

3.2.3 Maxwellovi svici

Sustav od tri kružna svitka, dva jednaka vanjska i jedan veći unutarnji, naziva se Maxwellovim svicima (slika 10). U literaturi se vrlo rijetko spominju u usporedbi s Helmholtzovim svicima, unatoč postizanja bolje homogenosti (slika 9).

Na slici 9 prikazana je relativna promjena indukcije B_z od ishodišta u smjeru osi z iskazanog omjerom $\xi = z/a$ za tri sustava s kružnim svicima: Helmholtzovi svici , Maxwellovi svici te poboljšani Helmholtzovi svici [19].

Maxwellovi svici su serijski spojeni i njima teče

A comparison of the improved Helmholtz coils with the ordinary ones, as well as with the Maxwell coils, is graphically represented in Figure 9 [19], where $a = (B_z - B_0) / B_0$ relative induction deviation along axis *z* from induction in the center of the system (homogeneity), and the distance on axis *z* is expressed by ratio $\xi = z/a$. A significant improvement has been achieved across a wider range (curves 1 and 3 compared).

The basic systems with at least three pairs of the Helmholtz coils are also used for more complex tests at established laboratories, in order to get a homogeneous magnetic field in three directions.

3.2.3 Maxwell coils

The system of three circular coils, two identical outer coils and a larger inner coil, is referred to as the Maxwell coils (Figure 10). Compared with the Helmholtz coils, they are rarely mentioned in literature in spite of the fact that they attain better homogeneity (Figure 9).

Figure 9 shows a relative change in induction B_z from the starting point in the direction of axis *z* expressed by the ratio $\xi = z/a$ for three systems with circular coils: the Helmholtz coils, the Maxwell coils, and the improved Helmholtz coils [19].



struja *I*. Polumjer srednjeg svitka je *a*, a polumjer svakog od manjih svitaka je $a_1 = (4/7)^{1/2} \cdot a = 0,756 \cdot a$. Svaki od manjih svitaka udaljeni su od većeg za $d = (3/7)^{1/2} \cdot a = 0,655 \cdot a$. Veći svitak ima 64 zavoja, a manji svaki po 49 zavoja. Magnetska indukcija u bilo kojoj točki na z osi sustava je [19]: The Maxwell coils are serially connected and electric current *I* flows through them. The radius of the middle coil is *a*, and the radius of each of the smaller coils is $a_1 = (4 / 7)^{1/2} \cdot a = 0,756 \cdot a$. The distance of each of the smaller coils from the larger one is $d = (3 / 7)^{1/2} \cdot a = 0,655 \cdot a$. The larger coil has 64 turns, and the smaller ones 49 turns each. Magnetic induction in any point on *z* axis of the system is [19]:

Indukcija u središtu sustava, tj. u z = 0 je:

$$B_{z} = \frac{1}{2}\mu_{0}Ia^{2}\left\{64\left(a^{2} + z^{2}\right)^{\frac{3}{2}} + 28\left[\frac{4}{7}a^{2} + \left(z + \sqrt{\frac{3}{7}}a\right)^{2}\right]^{-\frac{3}{2}} + 28\left[\frac{4}{7}a^{2} + \left(z - \sqrt{\frac{3}{7}}a\right)^{2}\right]^{-\frac{3}{2}}\right\}.$$
 (15)

Induction in the center of the system, i.e., in z = 0 is:

$$B_0 = 60 \frac{\mu_0 I}{a} . (14)$$



Slika 10 — Maxwellovi svici s tri kružna svitka Figure 10 — Maxwell coils with three circular coils

4 ZAKLJUČAK

Magnetske indukcije reda veličine 10-2 T ili manie mogų se ostvariti različitim vrstama svitaka ili sustava svitaka kojima teče električna struja. Odabir svitka ili sustava svitaka ovisi o opsegu i vrsti objekta na koji se magnetsko polje primjenjuje. Općenito vrijedi da se to bolja homogenost magnetskog polja postiže u što manjem opsegu u usporedbi s onim čitavog svitka ili sustava svitaka. Uz isti broj zavoja svitaka i struje kroz njih veća se indukcija postiže valjkastim svicima. Poboljšanja u konstrukciji i izvedbi temeljnih svitaka ili sustava svitaka omogućuju postizanje relativne homogenosti u malom opsegu, čak reda veličine 10⁻⁷ i nesigurnosti reda veličine 10⁻⁶. Istraživania su pokazala da su četverokutni kratki svici bolji od kružnih, a Helmholtzovi i Maxwellovi svici, što se tiče homogenosti, bolji od kratkih svitaka.

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Zahvaljujem se lvici Kunštu dipl. ing. na trudu pri izradi slika.

4 CONCLUSION

Magnetic inductions of 10⁻²T order of magnitude or less can be attained with various types of coils or coil systems through which electric current flows. The choice of the coil or coil system depends on the range and type of objects to which a magnetic field is applied. It is generally held that the best possible homogeneity of magnetic field is achieved within the narrowest possible range compared with that of the whole coil or coil system. Given the same number of the turns of wire and electric currents flowing through them, greater induction is achieved with roller coils. Improvements in the design of basic coils or coil systems make it possible to achieve relative homogeneity across a small range, even of 10^{-7} order of magnitude and uncertainty of 10⁻⁶ order of magnitude. Research studies have shown that the short square coils are better than the circular ones and the Helmholtz and Maxwell coils, when it comes to homogeneity, better than the short coil.

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